**Simple Origami Problem**

You are given a square paper of sides 24.

E is the mid-point of BC.

The piece of paper is folded so that the point D coincides with point E.

HF is the line of folding and point A goes to point G.

By considering the ΔCEF, find all line segments in the diagram.

**Solution**

Let $CF=x$

 $CE=\frac{1}{2}×24=12$

 $EF=FD=24-x$

By Pythagoras Theorem,

 $\left(24-x\right)^{2}=x^{2}+12^{2}$

 $x^{2}-48 x+576=x^{2}+144$

 $∴x=9$

 $CF=x=9$

 $EF=FD=24-9=15$

 Note that $∠IEF=90°$

 $∠BEI=90°-∠CEF=∠CFE$

 Hence $∆BEI\~∆CFE (AAA)$

 $\frac{BE}{CF}=\frac{EI}{FE}=\frac{IB}{EC}⟹\frac{12}{9}=\frac{EI}{15}=\frac{IB}{12}⟹\left\{\begin{matrix}EI=20\\IB=16\end{matrix}\right.$

 Hence, $IA=24-IB=24-16=8$

 Let $GH=HA=y, HI=8-y$

 Also, $∆GHI\~∆BEI (AAA)$

 $\frac{GH}{BE}=\frac{HI}{EI}=\frac{IG}{IB}⟹\frac{y}{12}=\frac{8-y}{20}=\frac{IG}{16}$

 $\frac{y}{12}=\frac{8-y}{20}⟹y=3$

 Hence, $GH=HA=3, HI=8-3=5$

 Also, $\frac{3}{12}=\frac{IG}{16}⟹IG=4$

By Pythagoras Theorem,

 $HF^{2}=AD^{2}+\left(FD-HA\right)^{2}$

 $HF^{2}=24^{2}+\left(15-3\right)^{2}$

 $HF=12 \sqrt{5}$

**Yue Kwok Choy**

**13/7/2016**